Common vacuum conservation amplitude in the theory of the radiation of mirrors in two-dimensional space-time and of charges in four-dimensional space-time

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Abstract

The action changes (and thus the vacuum conservation amplitudes) in the proper-time representation are found for an accelerated mirror interacting with scalar and spinor vacuum fields in $1+1$ space. They are shown to coincide to within the multiplier e^2 with the action changes of electric and scalar charges accelerated in $3+1$ space. This coincidence is attributed to the fact that the Bose and Fermi pairs emitted by a mirror have the same spins 1 and 0 as do the photons and scalar quanta emitted by charges. It is shown that the propagation of virtual pairs in $1+1$ space can be described by the causal Green's function $\Delta_f(z,\mu)$ of the wave equation for $3+1$ space. This is because the pairs can have any positive mass and their propagation function is represented by an integral of the causal propagation function of a massive particle in $1+1$ space over mass which coincides with $\Delta_f(z,\mu)$. In this integral the lower limit μ is chosen small, but nonzero, to eliminate the infrared divergence. It is shown that the real and imaginary parts of the action change are related by dispersion relations, in which a mass parameter serves as the dispersion variable. They are a consequence of the same relations for $\Delta_f(z,\mu)$. Therefore, the appearance of the real part of the action change is a direct consequence of the causality, according to which $\mathrm{Re}\,\Delta_f(z,\mu)\neq 0$ only for timelike and zero intervals.

1. INTRODUCTION

An intriguing symmetry between the creation of particle pairs by an accelerated mirror in $1+1$ space and the emission of single quanta by a charge accelerated as a mirror in $3+1$ space was discovered in Refs. 1–3. This symmetry is confined to coincidence of the spectra of the Bose and Fermi pairs created by the mirror with the spectra of the photons and scalar quanta emitted by electric and scalar charges, if the doubled frequencies ω and ω' of the quanta in a pair created by the mirror are identified with the components $k_\pm = k^0 \pm k^1$ of the 4-wave vector k^α of the quantum emitted by the charge:

$$2\omega = k_+, \quad 2\omega' = k_-. \tag{1}$$

It was shown in Ref. 3 that the Bogolyubov coefficients $\beta^B_{\omega'\omega}$ and $\beta^F_{\omega'\omega}$, which describe the spectra of Bose and Fermi radiation of a mirror, are related to the Fourier transforms of the 4-current density $j_\alpha(k_+, k_-)$ and the scalar charge density $\rho(k_+, k_-)$, which describe the spectra of photons and scalar quanta emitted by charges, by the following expressions[1]

$$\beta^{B*}_{\omega'\omega} = -\sqrt{\frac{k_+}{k_-}}\frac{j_-}{e} = \sqrt{\frac{k_-}{k_+}}\frac{j_+}{e}, \tag{2}$$

$$\beta^{F*}_{\omega'\omega} = \frac{1}{e}\rho(k_+, k_-). \tag{3}$$

It was also shown that $\beta^*_{\omega'\omega}$ is the source amplitude of a pair of particles which are only potentially emitted to the right and to the left with the frequencies ω and ω'. In other words, it is the virtual-pair creation amplitude. The pair becomes real when one of its particles undergoes internal reflection with a frequency change and both particles move in the same direction, i.e., to the right in the case of a right-sided mirror and to the left in the case of a left-sided mirror. Therefore, for a right-sided mirror, for example, the emission amplitude $\langle out\, \omega''\omega | in\rangle$ of a real pair of particles with the frequencies ω and ω'' is connected with the virtual-pair creation amplitude $\beta^*_{\omega'\omega}$ by the relation

$$\langle out\, \omega''\omega | in\rangle = -\sum_{\omega'} \langle out\, \omega'' | \omega'\, in\rangle \beta^*_{\omega'\omega}, \tag{4}$$

where $\langle out\, \omega''|\omega'\, in\rangle$ is the amplitude of single-particle scattering on the mirror. The energy and momentum of this real pair equal $\omega + \omega''$ and $\omega + \omega''$, i.e., the pair does not have mass, nor do its components.

A virtual pair is another matter. According to (1), the zeroth and first components of the 4-momentum k^α of a quantum emitted by a charge are equal to the energy and momentum of a virtual pair of massless particles created by a mirror:

$$k^0 = \omega + \omega', \quad k^1 = \omega - \omega', \tag{5}$$

and form the timelike 2-momentum of the pair in $1+1$ space. Clearly, the quantity

$$m = \sqrt{k_+ k_-} = 2\sqrt{\omega\omega'}, \tag{6}$$

being an invariant of Lorentz transformations along axis 1, is the mass of the virtual pair, and, at the same time, it equals the transverse momentum $k_\perp = \sqrt{k_2^2 + k_3^2}$ of the massless real quantum emitted by the charge.

The fact that the source amplitude $\beta^B_{\omega'\omega}$ of a virtual pair of bosons is specified by the current $j^\alpha(k_+, k_-)$, while the source amplitude $\beta^F_{\omega'\omega}$ of a virtual pair of fermions is specified by the scalar $\rho(k_+, k_-)$, means that the spin of a boson pair equals 1, while the spin of a fermion pair equals zero. Thus, the coincidence between the emission spectra of a mirror in $1+1$ space and charges in $3+1$ space can be attributed to the coincidence between the moment of a pair emitted by the mirror and the spin of a particle emitted by the charge.[3]

The relation (2) can be written in the explicitly invariant form

$$e\beta^{B*}_{\omega'\omega} = \varepsilon_{\alpha\beta} k^\alpha j^\beta / \sqrt{k_+ k_-}, \tag{7}$$

and, more specifically, in the form of the scalar product of the 2-current vector j^β and the 2-polarization pseudovector a_β of a Bose pair

$$a_\beta = \frac{\varepsilon_{\alpha\beta} k^\alpha}{\sqrt{k_+ k_-}}, \quad a_0 = -\frac{k^1}{\sqrt{k_+ k_-}}, \quad a_1 = \frac{k^0}{\sqrt{k_+ k_-}}. \tag{8}$$

The spacelike pseudovector a_β is constructed from the zeroth and first components of the 4-momentum k^α of the quantum emitted by the mirror. It is orthogonal to the 2-momentum of the pair, has a length equal to 1, and is represented in the comoving frame of the pair only by a spatial component, as is the current vector j^α.

In this work we find the vacuum conservation amplitude for acceleration of a mirror, which is defined by the change ΔW in the self-interaction of the mirror due to its acceleration. The problem here is essentially finding $\mathrm{Re}\,\Delta W$ from the previously found quantity $\mathrm{Im}\,\Delta W$, whose doubled value coincides in a certain approximation (see below) with the mean number of real pairs formed by the mirror. Three different methods are used for this purpose.

The first (and principal) method is considered in Sec. 2 and consists in transforming the original space-time representation for the mean number of pairs into a proper-time representation, whose kernel turns out to be the relativistically invariant singular even solution $(1/2)D^1(z)$ of the wave equation in $3+1$ space. Then, the function $D^1(z)$ in the expression obtained for the number of pairs is replaced by the even solution $\Delta^1(z,\mu)$ of the Klein–Gordon equation in order to invariantly and symmetrically eliminate the infrared divergence in the integral for the number of pairs using the small mass parameter μ instead of the large trajectory-length parameter L used in the original expression. The parameters $\mu, L^{-1} \ll \kappa$, if κ is the characteristic acceleration on the trajectory. Finally, treating the function $(1/2)\Delta^1(z,\mu)$ as the imaginary part of the kernel defining ΔW, by analytic continuation with respect to z^2 we can reconstruct a relativistically invariant and even in z kernel which coincides with the causal Green's function $\Delta_f(z,\mu)$ specific to $3+1$ space. The resultant action changes of a mirror and a charge differ only by the multiplier e^2, and the interactions are described by the same causal propagation function. Thus, the difference in dimensionality of the spaces is compensated by the difference in the mechanism of interaction transfer: it is realized by pairs in $1+1$ space and by individual particles in $3+1$ space.

Section 3 presents a direct calculation of the self-interaction changes $\Delta W_f^{B,F}$ for a concrete, but fairly general mirror trajectory. The invariant functions of the relative velocity of the trajectory ends obtained for $\Delta W_f^{B,F}$ are consistent with the results of Sec. 2.

In Sec. 4 $\mathrm{Re}\,\Delta W_f$ is reconstructed from $\mathrm{Im}\,\Delta W_f$ using dispersion relations, in which μ

appears as the dispersion variable. It is shown that the dispersion relations for ΔW_f are a consequence of the same relations for $\Delta_f(z,\mu)$ with timelike z as the parameter. As a consequence of causality only for such z the values of $\mathrm{Re}\,\Delta_f$ and $\mathrm{Re}\,\Delta W_f$ are nonzero and are connected with $\mathrm{Im}\,\Delta_f$ and $\mathrm{Im}\,\Delta W_f$, respectively, by the dispersion relations.

The fifth section examines other analytic continuations of $i\Delta^1/2$ onto the real z^2 axis that lead to kernels for ΔW whose real parts are not even in z.

A physical interpretation of the results is presented in the sixth, concluding section. The appearance of a causal propagation function characteristic for four-dimensional space-time in two-dimensional space-time is attributed to interaction transfer by pairs of different mass.

2. PROPER-TIME REPRESENTATION OF THE CHANGE OF ACTION

The following representations were obtained in Ref. 2 for mean numbers of radiated Bose and Fermi particles:

$$N^{B,F} = \frac{1}{4\pi^2}\int\limits_{-\infty}^{\infty} du\, K^{B,F}(u), \tag{9}$$

$$K^B(u) = \fint\limits_{-\infty}^{\infty} \frac{dv}{v-f(u)}\left[\frac{1}{g(v)-u} - \frac{f'(u)}{v-f(u)}\right], \tag{10}$$

$$K^F(u) = -\sqrt{f'(u)}\int\limits_{-\infty}^{\infty} \frac{dv}{v-f(u)}\left[\frac{\sqrt{g'(v)}}{g(v)-u} - \frac{\sqrt{f'(u)}}{v-f(u)}\right]. \tag{11}$$

It follows from these representations for trajectories with the asymptotically constant velocities β_1 and β_2 at the ends and a nonzero Lorentz-invariant relative velocity

$$\beta_{21} = \frac{\beta_2-\beta_1}{1-\beta_2\beta_1}, \quad \theta = \tanh^{-1}\beta_{21}, \tag{12}$$

that the mean number of massless quanta emitted is infinite (there is infrared divergence). In fact, in this case it follows from formulas (10) and (11) for $u \to \pm\infty$ (more precisely, for

$|u| \gg \kappa^{-1}$, i.e., outside the region where the mirror experiences the characteristic acceleration κ) that the functions $K^{B,F}(u)$ possess universal behavior, which depends only on β_{21}:

$$K^B(u) \approx \pm\frac{1}{u}\left(\frac{\theta e^{\mp\theta}}{\sinh\theta} - 1\right) = \pm\frac{1}{u}\left(\frac{\theta}{\tanh\theta} - 1\right) - \frac{\theta}{u}, \tag{13}$$

$$K^F(u) \approx \pm\frac{1}{u}\left(1 - \frac{\theta}{\sinh\theta}\right). \tag{14}$$

The relativistically invariant coefficients accompanying u^{-1} are formed on the parts of the trajectories with asymptotically constant velocities. As a result, the mean number of quanta emitted on the part of the trajectory covering the acceleration region grows logarithmically as the length $2L$ of that part is increased:

$$N^B = \frac{1}{2\pi^2}\left(\frac{\theta}{\tanh\theta} - 1\right)\ln(L\kappa) + 2b^B(\theta), \tag{15}$$

$$N^F = \frac{1}{2\pi^2}\left(1 - \frac{\theta}{\sinh\theta}\right)\ln(L\kappa) + 2b^F(\theta), \quad L\kappa \gg 1. \tag{16}$$

Let us focus our attention on the fact that the odd (with respect to both u and θ) term in the asymptotics of $K^B(u)$ does not make a contribution to the integral defining N^B. The terms $2b^{B,F}$ do not depend on L if $L\kappa \gg 1$, but they can depend on the specific form of the trajectories.

We note that there are representations for $N^{B,F}$ which differ from (9)–(11) by mirror symmetry, i.e., by the replacements $u \rightleftarrows v$ and $f(u) \rightleftarrows g(v)$. The integrands $K^{B,F}(v)$ defining them differ from $K^{B,F}(u)$, but are denoted below by the same letter, since they are values of the same functional taken for two mirror-symmetrical trajectories: $K(u) \equiv K[u; g]$ and $K(v) \equiv K[v; f]$. As $v \to \pm\infty$, $K^{B,F}(v)$ have asymptoticses which differ from (13) and (14) by the replacements $u \to v$ and $\theta \to -\theta$.

The vacuum conservation amplitude of an accelerated mirror is specified by the action change $\Delta W = W|_0^F$ (i.e., the difference between the actions for the accelerated and unaccelerated mirror) and has the form $\exp(i\Delta W)$, where $2\,\mathrm{Im}\,\Delta W = N$, if the interference effects in the creation of two or more pairs are neglected. We shall consider particle and antiparticle to be nonidentical; otherwise, in the same approximation $2\,\mathrm{Im}\,\Delta W = (1/2)N$ (see Ref. 3).

Now the main task is to find $\mathrm{Re}\,\Delta W$. For this purpose, we obtain a suitable representation for $\mathrm{Im}\,\Delta W$ and utilize relativistic-invariance and causality arguments.

Let us consider the space-time representation for N which was the direct "parent" of the representation (9)–(11) [see Ref. 2]. In this representation

$$N^B = \iint\limits_{-\infty}^{\infty} du\, dv\, S(u,v)|_0^F, \quad S(u,v) = \frac{1}{8\pi^2}\left[\frac{1}{(v-f(u)-i\varepsilon)\,(g(v)-u-i\delta)} + \text{c.c.}\right]. \qquad (17)$$

We go over from the independent characteristic variables u and v to the proper-time moments τ and τ' of two points on the world trajectory of the mirror $x^\alpha(\tau)$:

$$u = x^0(\tau) - x^1(\tau) = x_-(\tau), \quad v = x^0(\tau') + x^1(\tau') = x_+(\tau'). \qquad (18)$$

Then

$$f(u) = x^0(\tau) + x^1(\tau) = x_+(\tau), \quad g(v) = x^0(\tau') - x^1(\tau') = x_-(\tau'), \qquad (19)$$

and $S(u,v)$ becomes a relativistically invariant function of the two-dimensional vector $z^\alpha = x^\alpha(\tau) - x^\alpha(\tau') \equiv (x-x')^\alpha$ joining the points $x^\alpha = x^\alpha(\tau)$ and $x'^\alpha = x^\alpha(\tau')$ on the mirror trajectory:

$$\begin{aligned} S(z) &= \frac{1}{8\pi^2}\left[\frac{1}{(x'_+ - x_+ - i\varepsilon)(x'_- - x_- - i\delta)} + \text{c.c.}\right] = \\ &= \frac{1}{8\pi^2}\left[\frac{1}{z_+z_- + i\varepsilon\,\mathrm{sgn}\,z^0} + \text{c.c.}\right] = \frac{1}{8\pi^2}\left[\frac{1}{-z^2 + i\varepsilon\,\mathrm{sgn}\,z^0} + \text{c.c.}\right] = -P\frac{1}{4\pi^2 z^2}. \qquad (20)\end{aligned}$$

The individual terms in (20) and their sum are well-known relativistically invariant singular functions in quantum electrodynamics (we use the notation in Ref. 4, but our D^1 and Δ^1 lack the multiplier i):

$$D^\pm(z) = \frac{\pm i}{4\pi^2(z^2 \pm i\varepsilon\,\mathrm{sgn}\,z^0)} = \frac{1}{4\pi^2}\left[\pi\varepsilon(z^0)\delta(z^2) \pm \frac{i}{z^2}\right],$$

(21)

$$D^1(z) = \frac{1}{2\pi^2 z^2},$$

so that

$$S(z) = -\frac{i}{2}\left[D^-(z) - D^+(z)\right] = -\frac{1}{2}D^1(z). \tag{22}$$

We stress that these functions are singular solutions of the wave equation in $3+1$ space, if z^α is construed as a four-dimensional, rather than a two-dimensional, vector. Here the appearance of these functions, which depend on the 2-vector z^α, is a result of the deep symmetry between the creation of a pair by a mirror in $1+1$ space and the emission of single quanta by a charge in $3+1$ space.

Using

$$\begin{aligned} du\, dv = d\tau\, d\tau' \dot{x}_- \dot{x}'_+ = d\tau\, d\tau' \left[\frac{1}{2}\left(\dot{x}_- \dot{x}'_+ + \dot{x}_+ \dot{x}'_-\right) + \frac{1}{2}\left(\dot{x}_- \dot{x}'_+ - \dot{x}_+ \dot{x}'_-\right)\right] = \\ = d\tau\, d\tau' \left(-\dot{x}_\alpha \dot{x}'^\alpha + \varepsilon_{\alpha\beta}\dot{x}^\alpha \dot{x}'^\beta\right) \end{aligned} \tag{23}$$

in the form of a sum of terms which are even and odd with respect to the interchange $\tau \rightleftarrows \tau'$ (a dot denotes differentiation with respect to the proper time), we obtain

$$N^B = \iint\limits_{-\infty}^{\infty} d\tau\, d\tau' \left(\dot{x}_\alpha \dot{x}'^\alpha - \varepsilon_{\alpha\beta}\dot{x}^\alpha \dot{x}'^\beta\right) \frac{1}{2} D^1(z)\Big|_0^F. \tag{24}$$

It is natural to use an explicitly relativistic method that conserves the symmetry relative to the interchange $\tau \rightleftarrows \tau'$ to eliminate the infrared divergence in (24). It consists of replacing the function $D^1(z)$ by the function $\Delta^1(z,\mu)$, which is also even in z and has the small mass parameter $\mu \ll \kappa$, where κ is the characteristic acceleration of the mirror.

This function

$$\frac{1}{2}\Delta^1(z,\mu) = \frac{\mu}{8\pi s}N_1(\mu s) = -\frac{1}{4\pi^2 s^2} - \frac{\mu}{4\pi^2 s}J_1(\mu s)\ln\frac{2}{\mu s} + R \tag{25}$$

(where J_1 and N_1 are Bessel and Neumann functions, and R is a regular function of s) is a singular solution of the wave equation in $3+1$ space, which depends only on the interval $s = \sqrt{-z^2}$ between the two points and preserves all the features with respect to s at $s = 0$.

It is called the Hadamard elementary function or the fundamental solution[5]. The coefficient in front of the logarithm, which is called the Riemann function[5], is a regular function of s, which satisfies the same equation as Δ^1. Just these two functions will define the imaginary and real parts of the action change.

Thus,

$$N^B = \iint\limits_{-\infty}^{\infty} d\tau\, d\tau' \left(\dot{x}_\alpha \dot{x}'^\alpha - \varepsilon_{\alpha\beta}\dot{x}^\alpha \dot{x}'^\beta\right) \frac{1}{2}\Delta^1(z,\mu)\Big|_0^F . \tag{26}$$

In the expressions for N^B the odd term is unessential, since $D^1(z)$ and $\Delta^1(z,\mu)$ are even relative to the replacement $z \to -z$.

Now regarding N as the imaginary part of the doubled action, it is naturally to consider the function $(1/2)\Delta^1(z,\mu)$ as imaginary part of some function $F(z^2)$, which is taken on the real z^2 axis, and which is analytic in the z^2 complex plane with a cut along the $z^2 \le 0$ semiaxis, where Lorentz invariance allows it to still depend on the sign of z^0, and coincides with $(i/2)\Delta^1(z,\mu)$ at $z^2 > 0$. Then the transition from iN^B to $2W$ is equivalent to the analytic continuation of $F(z^2)$ onto the real semiaxis $z^2 \le 0$. It is well known[6] that the boundary value of such a function, which does not depend on the sign of z^0 and is, therefore, even, is the limit from above ($\varepsilon \to +0$), which is called a causal function:

$$\Delta_f(z,\mu) = F(z^2 + i\varepsilon) = \frac{\mu}{4\pi^2 s} K_1(i\mu s) = \frac{1}{4\pi}\delta(s^2) - \frac{\mu}{8\pi s}\left[J_1(\mu s) - iN_1(\mu s)\right]. \tag{27}$$

Here K_1 is the McDonald function, and $s = \sqrt{-z^2 - i\varepsilon}$. The latter equality was written for $z^2 \le 0$, where $s \ge 0$ and Δ_f has a real part, which coincides with the Riemann function multiplied by $\pi/2$. If $z^2 > 0$, then $s = -i\sqrt{z^2}$, Δ_f is purely imaginary, and its imaginary part is positive.

Thus, for ΔW_f^B we obtain

$$\Delta W_f^B = \frac{1}{2}\iint d\tau\, d\tau' \dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau')\Delta_f(z,\mu)\Big|_0^F . \tag{28}$$

As was shown in Ref. 2, the space-time representation for N^F differs from the representation (17) for N^B by the additional multiplier $-\sqrt{f'(u)g'(v)}$ under the integral. Therefore, after the replacement of variables (18), instead of (23) we have

$$-du\,dv\sqrt{f'(u)g'(v)} = -d\tau\,d\tau'. \tag{29}$$

Then

$$N^F = \frac{1}{2}\iint d\tau\,d\tau'\Delta^1(z,\mu)\Big|_0^F, \tag{30}$$

and the action change equals

$$\Delta W_f^F = \frac{1}{2}\iint d\tau\,d\tau'\Delta_f(z,\mu)\Big|_0^F. \tag{31}$$

The proper-time representations obtained for $\Delta W_f^{B,F}$ differ from the self-action changes ΔW_1 and ΔW_0 of electric and scalar charges moving along the same trajectories as the mirror, but in $3+1$ space, only by the absence of the multiplier e^2.

At $\mu \to 0$, the coefficients in front of $\ln\mu^{-1}$ in the imaginary parts of the proper-time integrals (28) and (31) should coincide with the coefficients in front of $\ln L$ in the corresponding expressions for N^B and N^F [see (15) and (16)], since these coefficients cannot depend on the method used to eliminate the infrared divergence in the different representations for each of quantities N^B and N^F.

Since for an interval between two points on the timelike trajectory the

$$\mathrm{Re}\,\Delta_f(z,\mu) = -\frac{\mu}{8\pi s}J_1(\mu s) \tag{32}$$

and differs from the coefficient in front of the logarithm in $\mathrm{Im}\,\Delta_f$ only by the multiplier $\pi/2$ [see (27) and (25)], $\mathrm{Re}\,\Delta W_f$ also differs by the same multiplier from the coefficient in front of $\ln\mu^{-1}$ in $\mathrm{Im}\,\Delta W_f$. Thus, to within terms which vanish at $\mu \to 0$, we have

$$\Delta W_f = \pi a(\theta) + i\left[a(\theta)\ln\frac{\kappa^2}{\mu^2} + b(\theta)\right], \tag{33}$$

$$a^B(\theta) = \frac{1}{8\pi^2}\left(\frac{\theta}{\tanh\theta} - 1\right), \quad a^F(\theta) = \frac{1}{8\pi^2}\left(1 - \frac{\theta}{\sinh\theta}\right). \tag{34}$$

The function $b(\theta)$ can depend on other dimensionless parameters, for example, the velocity changes on parts of the trajectory containing other extreme values of the proper acceleration.

It is significant that for $\theta \neq 0$ the $\mathrm{Re}\,\Delta W_f = \pi a$ has a finite positive limit at $\mu \to 0$.

To conclude this section we recall that $\mathrm{Re}\,\Delta W_f$ is the acceleration-induced self-energy shift of the source integrated over the proper time and that $2\,\mathrm{Im}\,\Delta W_f$ is the mean number of emitted pairs (or emitted particles in the case of their nonidentity to the antiparticles). More precisely, $\exp(-2\,\mathrm{Im}\,\Delta W_f)$ is the probability of the noncreation of pairs during the all time of acceleration.

3. ACTION CHANGE IN THE CASE OF QUASIHYPERBOLIC MOTION OF MIRROR

It would be interesting to directly calculate $\Delta W_f^{B,F}$ for the special, but very important mirror trajectory

$$x = \xi(t) = v_\infty \sqrt{\frac{v_\infty^2}{\kappa^2} + t^2}, \tag{35}$$

which can be called quasihyperbolic. Here $\pm v_\infty$ are the velocities of the mirror at $t \to \pm\infty$, and κ is its acceleration at the turning point ($t = 0$). This motion is remarkable in that as $v_\infty \to 1$, it becomes increasingly close to uniformly accelerated (hyperbolic) motion over the increasingly longer time interval

$$|t| \lesssim t_1 = \frac{v_\infty}{\kappa}(1 - v_\infty^2)^{-1/2},$$

smoothly going over to uniform motion outside this interval. This can be seen from the expression for the magnitude of the acceleration in the proper frame

$$a = \kappa \left(1 + \frac{t^2}{t_1^2}\right)^{-3/2}.$$

The spectrum and total radiated energy were found for an electric charge moving along the trajectory (35) in Ref. 7.

To calculate ΔW^B, in (28) instead of t we use the variable u, which is defined by the formula

$$t = \frac{v_\infty}{\kappa} \sinh u.$$

Then

$$d\tau\, d\tau' \dot{x}_\alpha(\tau) \dot{x}^\alpha(\tau') = -dx\, dy \frac{v_\infty^2}{\kappa^2} \left(\frac{1+v_\infty^2}{2} \cosh x + \frac{1-v_\infty^2}{2} \cosh 2y \right),$$

$$(36)$$

$$(x-x')^2 = -2\frac{v_\infty^2}{\kappa^2} (\cosh x - 1) \left[v_\infty^2 + (1-v_\infty^2) \cosh^2 y \right], \quad x = u - u', \quad y = \frac{u+u'}{2},$$

and ΔW^B is expressed by the integral of the McDonald function

$$\Delta W^B = -\frac{1}{32\pi^2} \int\limits_0^\infty \frac{d\xi}{\xi^2} e^{-i\mu^2 \xi} \int\limits_{-\infty}^\infty dy\, \frac{v_\infty^2}{\kappa^2} e^{iz} \left[(1+v_\infty^2) K_1(iz) + (1-v_\infty^2) K_0(iz) \cosh 2y \right]_0^F, \quad (37)$$

if we use the representation

$$\Delta_f(x-x', \mu) = \frac{1}{16\pi^2} \int\limits_0^\infty \frac{d\xi}{\xi^2} \exp\left[i\frac{(x-x')^2}{4\xi} - i\mu^2 \xi \right] \quad (38)$$

for the causal function and introduce the notation

$$z = \frac{v_\infty^2}{2\kappa^2 \xi} \left[v_\infty^2 + (1-v_\infty^2) \cosh^2 y \right]. \quad (39)$$

Now going over from the integration variable ξ to z in (37), we obtain

$$\Delta W^B = -\frac{1}{8\pi^2} \int\limits_{-\infty}^\infty dy \left\{ \frac{1+v_\infty^2}{2Q} \left[S_1(\Lambda) + S_0(\Lambda) \right] - S_0(\Lambda) \right\}, \quad (40)$$

where

$$\Lambda = \lambda v_\infty^2 Q, \quad \lambda = \frac{\mu^2}{\kappa^2}, \quad Q = v_\infty^2 + (1-v_\infty^2) \cosh^2 y, \quad (41)$$

$$S_n(\Lambda) = (-1)^{n+1} \int\limits_0^\infty dz\, e^{-i\Lambda/2z} \left[e^{iz} K_n(iz) - \sqrt{\frac{\pi}{2iz}} \right]. \quad (42)$$

The subtraction $|_0^F$ in (40) was reduced to subtraction of the asymptotics $(\pi/2iz)^{1/2}$ of the integrand in (42) for $S_n(\Lambda)$. As was shown in Refs. 8 and 9, the functions $S_n(\Lambda)$ are expressed in terms of the product of the modified Bessel functions $I_n(\sqrt{\Lambda})$ and $K_n(\sqrt{\Lambda})$. We also turn attention to the more compact expressions for the derivatives

$$S_n'(\Lambda) = (-1)^n \pi \left[I_n(x) K_n(x) - \frac{1}{2x} \right] + i K_n^2(x), \quad x = \sqrt{\Lambda}. \tag{43}$$

It can be seen from formulas (40)–(42) that ΔW^B depends on two dimensionless parameters, viz., λ and $v_\infty = \tanh(\theta/2)$.

To calculate the asymptotics of the integral (40) at $\lambda \to 0$ we note that in this case the values $\Lambda \to 0$ will be effective in the first term and, therefore,

$$S_1(\Lambda) + S_0(\Lambda) \approx -\pi - i \ln \frac{4}{\gamma^2 \Lambda}, \quad \gamma = 1.781\ldots, \tag{44}$$

and that in the second term the integral can be reduced to the expression

$$\begin{aligned} \int\limits_{-\infty}^{\infty} dy\, S_0(\Lambda) &\approx -\int\limits_0^{\infty} d\Lambda\, S_0'(\Lambda) \ln \Lambda + S_0(0) \ln \frac{4}{\lambda v_\infty^2 (1 - v_\infty^2)} = \\ &= -\pi - i \left[\ln \frac{16}{\gamma^2 v_\infty^2 (1 - v_\infty^2) \lambda} - 2 \right]. \end{aligned} \tag{45}$$

As a result, to within terms which vanish at $\lambda \to 0$ we obtain

$$\begin{aligned} \Delta W^B \approx \frac{1}{8\pi^2} \Bigg\{ \pi \left(\frac{\theta}{\tanh\theta} - 1 \right) + i \Bigg[\left(\frac{\theta}{\tanh\theta} - 1 \right) \ln \left[\frac{8(\cosh\theta + 1)^2}{\gamma^2 \lambda (\cosh\theta - 1)} \right] + \\ + 2 - \frac{L_2(1 - e^{-2\theta}) + \theta^2}{\tanh\theta} \Bigg] \Bigg\}. \end{aligned} \tag{46}$$

Here $\theta = \tanh^{-1} \beta_{21} = 2 \tanh^{-1} v_\infty$, and $L_2(x)$ is a Euler dilogarithm.[10,11]

For a quasiuniformly accelerated mirror interacting with a spinor field, instead of (40) we obtain

$$\Delta W^F = \frac{1}{8\pi^2} \int\limits_{-\infty}^{\infty} dy \left\{ \int\limits_0^{\infty} dz\, \exp\left(-\frac{i\Lambda}{2z} + iz \right) R(iz) - S_0(\Lambda) \right\}, \tag{47}$$

where

$$R(iz) = \int\limits_0^{\infty} dx \left(\sqrt{\frac{(\cosh x + c)^2 - s^2}{(1 + c)^2 - s^2}} - 1 \right) \exp(-iz \cosh x),$$

(48)

$$c = \cosh\theta\cosh 2y, \quad s = \sinh\theta\sinh 2y, \quad \theta = 2\tanh^{-1} v_\infty,$$

and the remaining notation is the same as in (40). It is seen that ΔW^F depends on the two dimensionless parameters λ and θ.

When $\lambda \to 0$, the expression in the large parentheses in (48) can be replaced by

$$\frac{\cosh x - 1}{\sqrt{(1+c)^2 - s^2}}.$$

This approximation holds for $\cosh x \gg 1$ and has the correct (zero) value at $x = 0$. Then

$$\Delta W^F \approx \frac{1}{8\pi^2}\int\limits_{-\infty}^{\infty} dy \left\{ \frac{1}{\sqrt{(1+c)^2 - s^2}} \left[S_1(\Lambda) + S_0(\Lambda)\right] - S_0(\Lambda) \right\}, \tag{49}$$

and using (44) and (45), we obtain

$$\Delta W^F \approx \frac{1}{8\pi^2}\left\{\pi\left(1-\frac{\theta}{\sinh\theta}\right)+i\left[\left(1-\frac{\theta}{\sinh\theta}\right)\ln\left[\frac{8(\cosh\theta+1)^2}{\gamma^2\lambda(\cosh\theta-1)}\right]-2+\frac{L_2(1-e^{-2\theta})+\theta^2}{\sinh\theta}\right]\right\} \tag{50}$$

to within terms which vanish at $\lambda \to 0$.

The formulas obtained for $\Delta W^{B,F}$ not only have the structure (33), but also contain explicit expressions for $b^{B,F}(\theta)$. It can also be seen that $\Delta W^{B,F}$ do not depend on the sign of θ or β_{21}, if we take into account that $L_2(1-e^{-2\theta})+\theta^2$ is an odd function of θ [see Landen's formula (1.12) in Ref. 11]. We note in this connection that for small values of θ

$$L_2(1-e^{-2\theta}) + \theta^2 = 2\theta + \frac{2}{9}\theta^3 - \frac{2}{225}\theta^5 + \ldots, \tag{51}$$

and that as $\theta \to \pm\infty$, to within exponentially small terms we have

$$L_2(1-e^{-2\theta}) + \theta^2 = \pm\left(\theta^2 + \frac{\pi^2}{6}\right) + \ldots. \tag{52}$$

The imaginary and real parts of $\Delta W^{B,F}$ in (46) and (50) are positive owing to unitarity and causality. When $\theta = 0$, ΔW vanishes, since the quasihyperbolic trajectory becomes straight line.

The point $\theta = \infty$ for $\Delta W_f^{B,F}(\theta, \lambda)$ is essential singularity. It physically corresponds to a purely hyperbolic trajectory for which $\beta_{21} = 1$ or -1 in accordance with the sign of κ. At a fixed value of λ and $\theta \to \pm\infty$, from (40) and (47) we obtain

$$\Delta W_f^{B,F}(\theta, \lambda) = \mp\theta \frac{1}{8\pi^2} S_{1,0}(\lambda). \tag{53}$$

Here $\pm\theta = |\kappa|(\tau_2 - \tau_1) \gg 1$, and when the length of proper time interval (τ_1, τ_2) approaches to infinity the relative velocity β_{21} approaches $+1$ or -1. Formula (53) was obtained for uniformly accelerated charges in $3+1$ space in Ref. 12 and was discussed in detail in Refs. 8 and 9. In those studies it defined the classical mass shift of a uniformly accelerated charge:

$$\Delta m_{1,0} = -\frac{\partial \Delta W_{1,0}}{\partial \tau_2} = \frac{\alpha}{2\pi}|\kappa| S_{1,0}(\lambda). \tag{54}$$

In accordance with the unitarity and causality, the imaginary and real parts of Δm are negative. At $\kappa = 0$ the function $\Delta m(\kappa)$ is nonanalytic and, therefore, cannot be reproduced by perturbation theory with respect to κ or with respect to the field accelerating the charge.

4. DISPERSION RELATIONS FOR ΔW AND THEIR ORIGIN

It was shown in Ref. 9 that the action changes $\Delta W_s(\mu^2)$ of point charges moving along timelike trajectories as functions of the square of the mass of quanta of their proper-field with spin $s = 1, 0$ are analytic in the μ^2 complex plane with a cut along the positive μ^2 semiaxis, on whose edges the imaginary parts of each of the functions coincide, while the real parts differ in sign. Such functions satisfy the dispersion representations ($\mathrm{Im}\,\mu < 0$):

$$\Delta W(\mu^2) = \frac{2i}{\pi}\int\limits_0^\infty \frac{dx\, x\, \mathrm{Re}\, \Delta W(x^2)}{x^2 - \mu^2} = -\frac{2\mu}{\pi}\int\limits_0^\infty \frac{dx\, \mathrm{Im}\, \Delta W(x^2)}{x^2 - \mu^2}, \tag{55}$$

which reconstruct the function $\Delta W_s(\mu^2)$ in the μ^2 complex plane from its real or imaginary part assigned on the lower edge of the cut. When $\mu = i\kappa$ and $\kappa > 0$, these relations yield the important equalities

$$\frac{2}{\pi}\int\limits_0^\infty \frac{dx\, x\, \mathrm{Re}\, \Delta W(x^2)}{x^2 + \kappa^2} = \frac{2\kappa}{\pi}\int\limits_0^\infty \frac{dx\, \mathrm{Im}\, \Delta W(x^2)}{x^2 + \kappa^2} = \mathrm{Im}\, \Delta W(-\kappa^2) > 0, \tag{56}$$

$$\mathrm{Re}\,\Delta W(-\kappa^2) = 0. \tag{57}$$

As a consequence of unitarity, the $\mathrm{Im}\,\Delta W(\mu^2)$ is positive on the real semiaxis $\mu^2 > 0$. Then, according to the second of the representations (55), $\mathrm{Im}\,\Delta W(\mu^2)$ is positive definite over the entire μ^2 complex plane (or in the lower μ half-plane).

Here we show that the dispersion relations presented for $\Delta W(\mu^2)$ are due to the analytic properties of the causal Green's function $\Delta_f(z,\mu)$, which, as we see, specifies not only $\Delta W_s(\mu^2)$ for the vacuum amplitude of accelerated charges in 3+1 space, but also $\Delta W^{B,F}(\mu^2)$ for the vacuum amplitude of an accelerated mirror in $1+1$ space.

We can show that the causal function $\Delta_f(z,\mu)$ for a timelike z satisfies the dispersion relations presented. According to formulas (2.12.4.28) and (2.13.3.20) from Ref. 13

$$\int\limits_0^\infty \frac{dx\, x^2 J_1(sx)}{x^2+\kappa^2} = -\kappa \int\limits_0^\infty \frac{dx\, x N_1(sx)}{x^2+\kappa^2} = \kappa K_1(s\kappa), \tag{58}$$

where $s, \mathrm{Re}\,\kappa > 0$. After the analytic continuation in κ to the point $\kappa = i\mu + \varepsilon$ where $\mu > 0$ and $\varepsilon \to +0$ these relations turn into

$$\int\limits_0^\infty \frac{dx\, x^2 J_1(sx)}{x^2-\mu^2+i\varepsilon} = -i\mu \int\limits_0^\infty \frac{dx\, x N_1(sx)}{x^2-\mu^2+i\varepsilon} = i\mu K_1(i\mu s) = -\frac{i\pi\mu}{2}\left[J_1(\mu s) - iN_1(\mu s)\right]. \tag{59}$$

After multiplying by $-i/4\pi^2 s$, they form the first pair of dispersion relations (55), which, instead of $\Delta W(\mu^2)$, contain the causal function (27) with a timelike vector z^α, for which $s = \sqrt{-z^2} > 0$. For spacelike z^α the interval $s = -i\sqrt{z^2}$, and $\Delta_f(z,\mu)$ is purely imaginary.

After being multiplied by $-1/4\pi^2 s$, the original formulas (58) coincide with the second pair of the relations (56) with the replacement of $\Delta W(\mu^2)$ by $\Delta_f(z,\mu)$. The function appearing on the right-hand side of these relations

$$-\frac{\kappa}{4\pi^2 s} K_1(\kappa s) = \mathrm{Im}\,\Delta_f(z, -i\kappa), \tag{60}$$

unlike $\mathrm{Im}\,\Delta W(-\kappa^2)$, is negative. In addition,

$$\mathrm{Re}\,\Delta_f(z, -i\kappa) = 0, \tag{61}$$

as can be seen from (27). This property is a consequence of the causality, according to which $\mathrm{Re}\,\Delta_f(z,\mu) = 0$ outside the light cone, i.e., for spacelike z^α. In this case the argument of the McDonald function in (27) is real and positive. When we go over to timelike z^α and a purely imaginary negative $\mu = -i\kappa$, this argument remains real and positive, whence follows (61).

While satisfying the dispersion relations (55) and (56) with respect to the "dispersion" variable μ, the function $\Delta_f(z,\mu)$, unlike $\Delta W(\mu^2)$, still depends on the fixed as yet parameter s, which equals the invariant interval between the two points chosen on the mirror trajectory with the proper times τ and τ', i.e., on $s = s(\tau,\tau')$. Integrating the dispersion relations for Δ_f over τ,τ' with the weight $(1/2)\dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau')$ or $1/2$ and performing the subtraction procedure, we obtain the dispersion relations for ΔW^B or ΔW^F, if, of course, the familiar conditions for changing the order of integration over x and τ,τ' are satisfied.

Thus, the dispersion relations for $\Delta W(\mu^2)$ are a consequence of the dispersion relations for $\Delta_f(z,\mu)$.

If follows from (56) that if $\mathrm{Im}\,\Delta W(\mu^2)$ is bound at zero, then $\mathrm{Re}\,\Delta W(\mu^2)$ must vanish at $\mu \to +0$. If, on the other hand, at $\mu \to 0$ the $\mathrm{Im}\,\Delta W(\mu^2)$ logarithmically tends to infinity according to the relation

$$\mathrm{Im}\,\Delta W(\mu^2) = a\ln\mu^{-2} + b(\mu^2) \tag{62}$$

[$a > 0$, and $b(\mu^2)$ is bound at zero], it follows from (56) that $\mathrm{Re}\,\Delta W(\mu^2)$ tends to the positive value $\mathrm{Re}\,\Delta W(0) = \pi a$ at $\mu \to 0$. According to (57), this means that the function $\mathrm{Re}\,\Delta W(\mu^2)$ has a discontinuity equal to πa on the real μ^2 axis at $\mu^2 = 0$.

5. INFLUENCE OF THE BOUNDARY CONDITIONS ON $\mathrm{Re}\,\Delta W$

Let us now consider the other boundary values of $F(z^2)$, which is analytic in the z^2 complex plane with a cut along the $z^2 \le 0$ semiaxis and coincides with $(i/2)\Delta^1(z,\mu)$ on the $z^2 > 0$ semiaxis.

The limit $F(z^2 - i\varepsilon)$ on the real axis from below is distinguished from the limit (27) from above by the opposite sign of the real part. According to this function, free fields would transfere negative energy in $3+1$ space; therefore, this boundary condition is not considered here.

The other boundary values of $F(z^2)$, which already depend on the sign of z^0, may be the limits $F(z^2 \pm i\varepsilon \,\mathrm{sgn}\, z^0)$, $\varepsilon \to +0$. They are positive- and negative-frequency functions, or, more precisely, $\pm\Delta^{\pm}(z,\mu)$ (Ref. 4):

$$\pm\Delta^{\pm}(z,\mu) = \pm\varepsilon(z^0)\,\mathrm{Re}\,\Delta_f + i\,\mathrm{Im}\,\Delta_f. \tag{63}$$

Such functions naturally vary only the real part of the action obtained for Δ_f, so that

$$\mathrm{Re}\,\Delta W^B_{\pm} = \pm\frac{1}{2}\iint d\tau\, d\tau' \left(\dot{x}_\alpha \dot{x}'^\alpha - \varepsilon_{\alpha\beta}\dot{x}^\alpha \dot{x}'^\beta\right) \mathrm{Re}\,\Delta^{\pm}(z,\mu)\Big|_0^F \tag{64}$$

differ from $\mathrm{Re}\,\Delta W^B_f$ and are given in the limit $\mu \to 0$ by the expressions

$$\mathrm{Re}\,\Delta W^B_{\pm} = \mp\frac{1}{8\pi}\iint d\tau\, d\tau' \varepsilon_{\alpha\beta}\dot{x}^\alpha \dot{x}'^\beta \varepsilon(z^0)\delta(z^2). \tag{65}$$

The integrand can be expanded in τ' near $\tau' = \tau$ and represented in the form

$$\varepsilon_{\alpha\beta}\dot{x}^\alpha \dot{x}'^\beta \varepsilon(z^0)\delta(z^2) = -\varepsilon_{\alpha\beta}\dot{x}^\alpha \ddot{x}^\beta \delta(\tau - \tau'). \tag{66}$$

Here the equality $|x|\delta(x^2) = \delta(x)$ was used (see, for example, Ref. 14).

Then, integrating over τ' and expressing the proper-acceleration

$$a(\tau) = \varepsilon_{\alpha\beta}\dot{x}^\alpha \ddot{x}^\beta = \frac{f''}{2(f')^{3/2}} = \frac{d\ln f'(u)}{2d\tau} = \frac{d\tanh^{-1}\beta(\tau)}{d\tau} \tag{67}$$

in the form of the derivative of the rapidity with respect to the proper time, we obtain

$$\mathrm{Re}\,\Delta W^B_{\pm} = \pm\frac{1}{8\pi}\int\limits_{-\infty}^{\infty} d\tau\, \varepsilon_{\alpha\beta}\dot{x}^\alpha \ddot{x}^\beta = \pm\frac{1}{8\pi}\tanh^{-1}\beta_{21} = \pm\frac{\theta}{8\pi}. \tag{68}$$

Clearly,

$$\mathrm{Re}\,\Delta W^F_{\pm} = \pm\frac{1}{2}\iint d\tau\, d\tau'\,\mathrm{Re}\,\Delta^{\pm}(z,\mu) = 0 \tag{69}$$

because of the oddness of $\mathrm{Re}\,\Delta^{\pm}$ with respect to z.

The expressions obtained for $\mathrm{Re}\,\Delta W_{\mp}$ up to the multiplier $(8\pi)^{-1}$ coincide with the odd in θ coefficients for the terms proportional to u^{-1} and v^{-1} in the asymptotic expansions of $K(u)$ and $K(v)$, respectively [see (13) and (14) and the comment following Eq. (16)]. At the same time, $\mathrm{Re}\,\Delta W_f$ up to the same multiplier $(8\pi)^{-1}$ coincides with the even in θ coefficient for the term proportional to u^{-1} or v^{-1} in the asymptotic expansion of $K(u)$ or $K(v)$. We note that all these coefficients, as well as the functions $K(u)$ and $K(v)$ themselves, are formed without any involvement of the parameter L, which eliminates the infrared divergence of the space-time integrals (9) for the mean number of particles emitted.

Thus, information on the interaction contained in $K(u)$ and $K(v)$, which determine $\mathrm{Im}\,\Delta W$, is conveyed to $\mathrm{Re}\,\Delta W$ owing to the causality and the boundary conditions. In addition, $\mathrm{Re}\,\Delta W_f$ contains information on the interaction which propagates within the light cone, and $\mathrm{Re}\,\Delta W_{\pm}$ contains information on the interaction which propagates along the light cone and is therefore local owing to the timelike character of the trajectory.

As we know,[4] the half-sum of the retarded and advanced fields is the proper-field of the source, and their half-difference is the radiation field escaping to infinity. Since

$$\mathrm{Re}\,\Delta_f = \frac{1}{2}\left(\Delta^{\mathrm{ret}} + \Delta^{\mathrm{adv}}\right),$$

and

$$\mathrm{Re}\,\Delta^{+} = \frac{1}{2}\left(\Delta^{\mathrm{ret}} - \Delta^{\mathrm{adv}}\right),$$

$\mathrm{Re}\,\Delta W_f$ describes the self-energy shift of the source, and $\mathrm{Re}\,\Delta W_{+}$ describes the interaction with the radiation field, i.e., with real quanta. The boundary condition which eliminates the interaction with virtual quanta or pairs seems unnatural.

6. DISCUSSION AND PHYSICAL INTERPRETATION OF RESULTS

The proper-time representations for the changes in the self-interaction of a mirror upon acceleration in a two-dimensional vacuum of scalar and spinor fields can be considered the most significant results of this work. These representations coincided with the representations for the changes in the self-interaction of electric and scalar charges accelerated in four-dimensional space-time. In other words, both were found to be identical functionals of the source trajectory.

This coincidence, first, confirms the correctness of the interpretation given in Ref. 3 of the Bogolyubov coefficient $\beta^*_{\omega'\omega}$ as the source amplitude of a virtual pair of particles potentially emitted to the right and to the left with the frequencies ω and ω', with the timelike 2-momentum of the pair (5), the mass $m = 2\sqrt{\omega\omega'}$, and a spin equal to 1 for a boson pair and 0 for a fermion pair.

Second, it means that the self-interaction of mirror is realized by the creation and absorption of virtual pairs, rather than individual particles, and is transfered from one point of the trajectory to another by the causal Green's function of the wave equation for four-dimensional, rather than two-dimensional space-time.

The action integral is formed by virtual pairs with the mass $m = 2\sqrt{\omega\omega'}$, which takes any positive values. Therefore, it is natural to expect that the effective propagation function of such pairs will be the integral of the propagation function of a massive particle in two-dimensional space-time over the mass m.

At the same time, it can be shown that the causal Green's functions for spaces of dimensionalities d and $d+2$, being functions of the invariant interval $s = \sqrt{-z^2}$ between two points and the mass μ, are related to one another by the equalities

$$\Delta_f^{(d+2)}(z,\mu) = \frac{1}{\pi}\frac{\partial}{\partial s^2}\Delta_f^{(d)}(z,\mu) = \frac{1}{4\pi}\int\limits_{\mu^2}^{\infty} dm^2\,\Delta_f^{(d)}(z,m) \tag{70}$$

and are expressed in terms of the McDonald function with the index specified by the dimensionality of the space-time:

$$\Delta_f^{(d)}(z,\mu) = \frac{i\mu^{2\nu}}{(2\pi)^{\nu+1}(i\mu s)^{\nu}}K_\nu(i\mu s), \quad \nu = \frac{d-2}{2}. \tag{71}$$

The second equality in (70) for $d = 2$ confirms the appearance of a causal function characteristic of four-dimensional space-time as an effective propagation function of virtual pairs with different masses m in two-dimensional space-time. Now the small mass parameter μ, which was introduced in Sec. 2 to eliminate the infrared divergence, can be interpreted as the lower bound of the masses of the virtual pairs transfering the self-interaction of a mirror.

A virtual pair can not escape to infinity, since one of its particles unavoidably undergoes reflection from the mirror, after which the pair becomes real and massless. The emission of such pairs forms $\mathrm{Im}\,\Delta W$. Owing to masslessness, the emission of arbitrary large number of arbitrary soft quanta becomes possible on trajectories with $\beta_{21} \neq 0$, i.e., infrared divergence of $\mathrm{Im}\,\Delta W_f$ appears. By choosing a nonzero, but sufficiently small value for μ, we eliminate the infrared divergence in $\mathrm{Im}\,\Delta W_f$ and make sure that $\mathrm{Re}\,\Delta W_f$ does not depend on μ at $\mu \ll |\kappa|$. This means that the main contribution to $\mathrm{Re}\,\Delta W_f$ is made by virtual pairs with a mass of the order of $|\kappa|$.

In the general case, where the mean number of pairs created is not small compared to 1, the quantity $2\,\mathrm{Im}\,\Delta W$ is no longer equal to the mean number of pairs $\mathrm{tr}(\beta^+\beta)$. Because of the interference of two or more being created pairs it equals

$$2\,\mathrm{Im}\,\Delta W = \pm \mathrm{tr}\ln(1 \pm \beta^+\beta)\big|_0^F = \pm \mathrm{tr}\ln(\alpha^+\alpha)\big|_0^F. \tag{72}$$

The last formula prompted De Witt[15] to consider the following expression for W natural:

$$W = \pm i\,\mathrm{tr}\ln\alpha. \tag{73}$$

The matrix formulation of the Bogolyubov coefficients α and β was adopted. In addition, tr must be replaced by $(1/2)$tr in the case of an identical particle and antiparticle[3].

We do not know of any concrete results for $\mathrm{Re}\,\Delta W$ emanating from (73).

The symmetry discussed would be total, if the equality $e^2 = \hbar c$ would hold in Heaviside units.

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